CHAPTER 4

RESULTS AND DISCUSSION

4.1 Analysis of the Error on One Term Approximation

One dimensional transient temperature was calculated for plane wall, long cylinder and solid sphere. Results of exact solution and one term approximation solution have been compared. One term approximation solution and exact solution have been investigated for values of Biot number which are 1, 2, 3, 4, 5, 6, 7, 7, 8, 9, 10, 20, 30, 40, 50 and 100 and for values of dimensionless time which are 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.1 0.15, 0.2 and 0.25. Dimensionless positions for plane wall has been accepted with interval 0.1 from -1 to 1 and for long cylinder and solid sphere have been accepted with interval 0.05 from 0 to 1. Successive values of λ have been found by using goal seek building feature of Microsoft Excel program. In calculation of exact dimensionless temperature value, no limit is applied to the number of terms of the infinite series. The series converges to zero for to higher values of λ. Therefore the total number of term for dimensionless temperature is automatically determined by the program. They have been shown in appendices for each geometry and for values of Biot number which are 1, 10, 50 and 100. Errors in two solutions have been especially researched for dimensionless time less than 0.2.

Error between two solutions has been defined as follows;

$$\mathcal{E} = \frac{\theta_{exact} - \theta_{one\,term}}{\theta_{exact}} 100 \tag{4.1}$$

Difference between exact solution and one term approximation solution has been shown for each geometry in Figure 4.1, Figure 4.2 and Figure 4.3. Difference has been seen more in centre of body for plane wall, long cylinder and solid sphere.

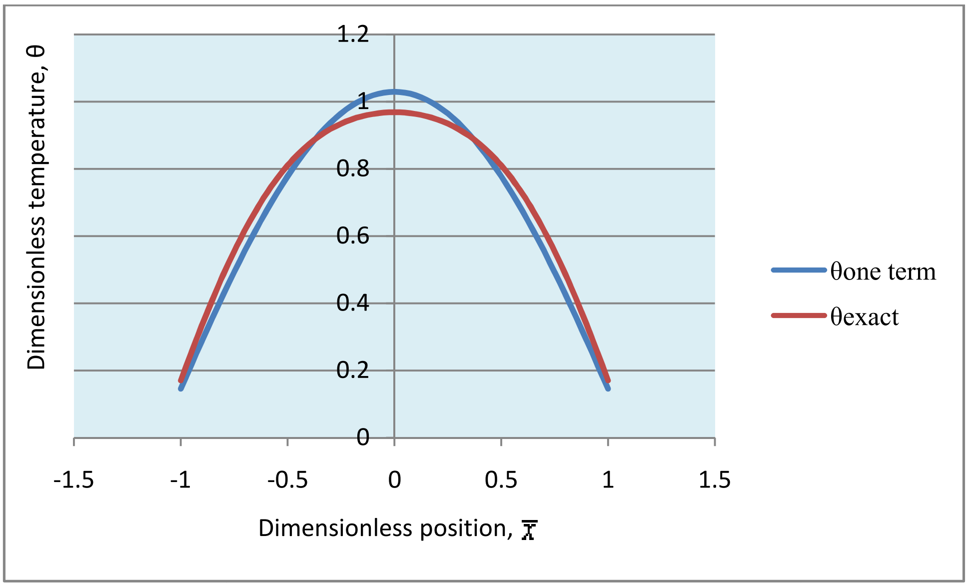

Figure 4.1: Difference between exact solution and one term approximation solution for plane wall for Bi = 10 and τ = 0.1

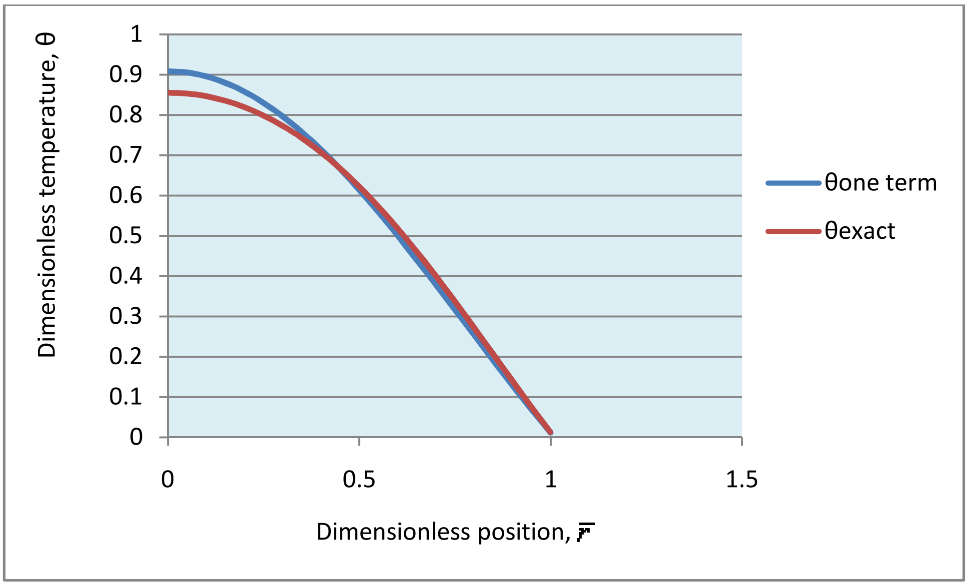

Figure 4.2: Difference between exact solution and one term approximation solution for cylinder for Bi = 10 and τ = 0.1

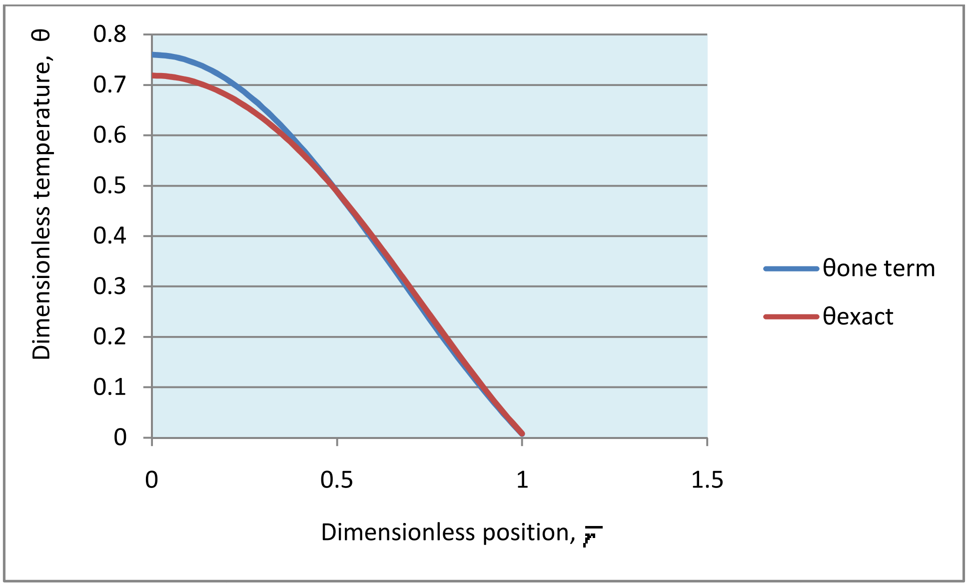

Figure 4.3: Difference between exact solution and one term approximation solution for sphere for Bi = 10 and τ = 0.1

4.1.1. Analysis of the Error for Plane Wall

Variation of errors figures have been shown for Bi=1, Bi=10 and Bi=100. Errors are small in centre of body, high outside. For small dimensionless time, variation of error is high. But for high dimensionless time, variation of error is small. Also for small values of Biot Number, variation of error is small. But for high values of Biot number, variation of error is high.

As can be seen from Figure 4.4 and Figures 4.5, for Bi=1; When it is $\tau = 0.005$, local error is about 25%. When it is $\tau = 0.2$, error is about 1%. For Bi=10; When it is $\tau = 0.005$, local error is about 60%. When it is $\tau = 0.2$, error is about 1%.

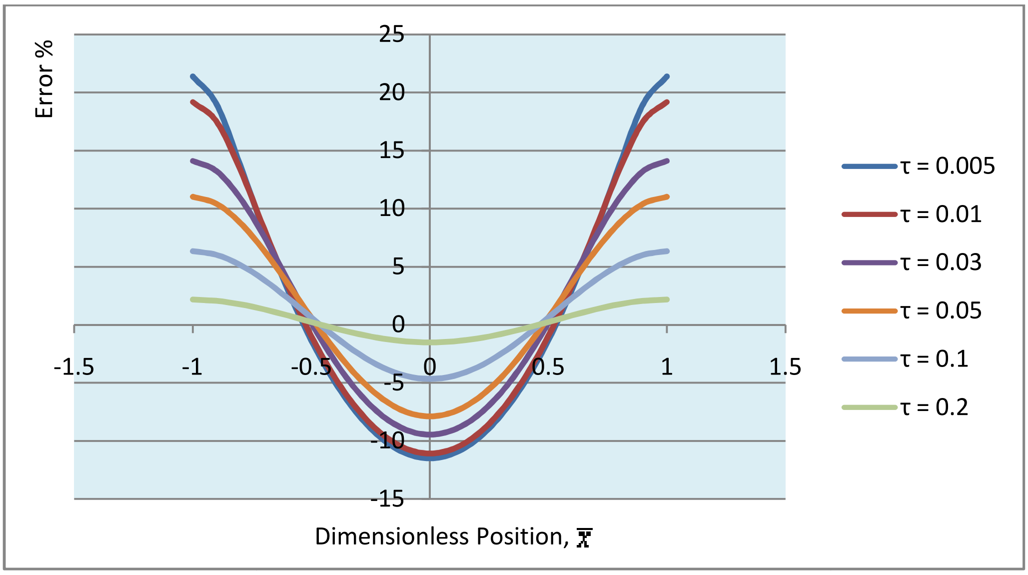

Figure 4.4: Variation of errors for plane wall for Bi = 1

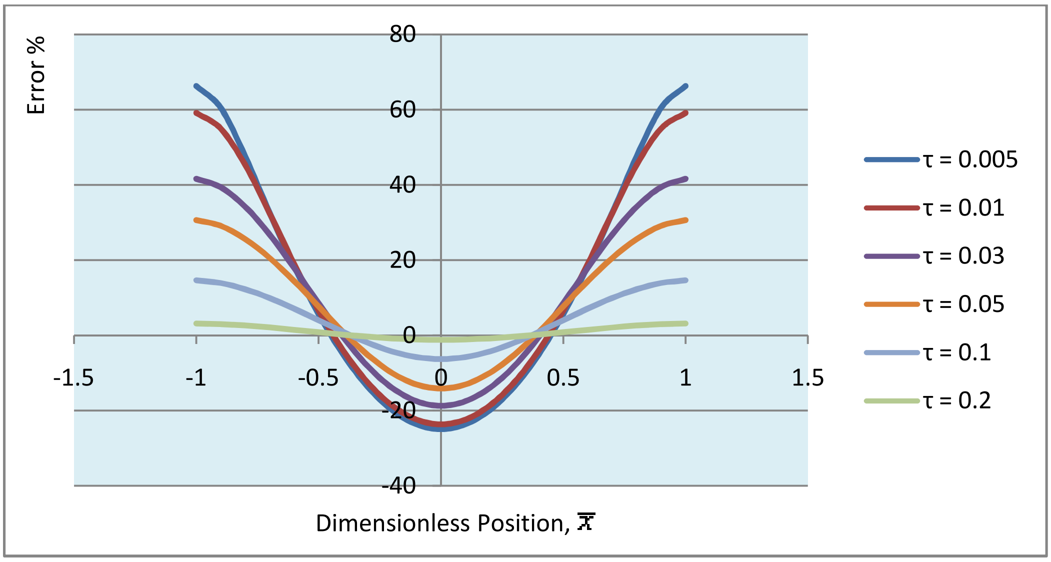

Figure 4.5: Variation of errors for plane wall for Bi = 10

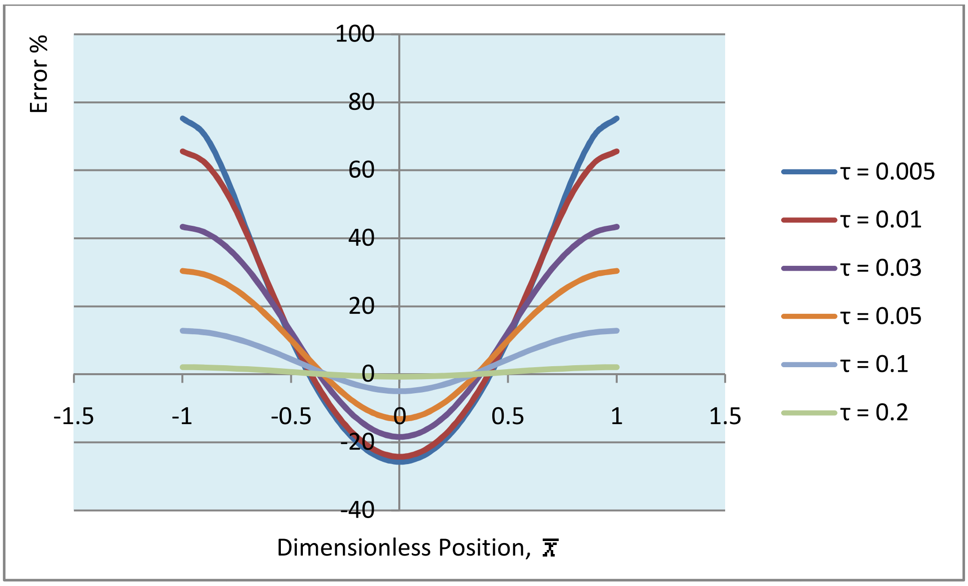

Figure 4.6: Variation of errors for plane wall for Bi = 100

4.1.2 Analysis of the Error for Long Cylinder

Variation of errors figures have been obtained for Bi=1, Bi=10 and Bi=100. For small dimensionless time, error is high. But for high dimensionless time, error is small. Also for small values of Biot number, error is small. But for high values of Biot number, error is high.

As can be seen from Figure 4.7 and Figures 4.8, for instance Bi = 1; When it is $\tau = 0.005$, local error is about 12%. When it is $\tau = 0.20$, local error is about 1%. For instance Bi=10; When it is $\tau = 0.005$, error is about 65%. When it is $\tau = 0.20$, local error is about 1%.

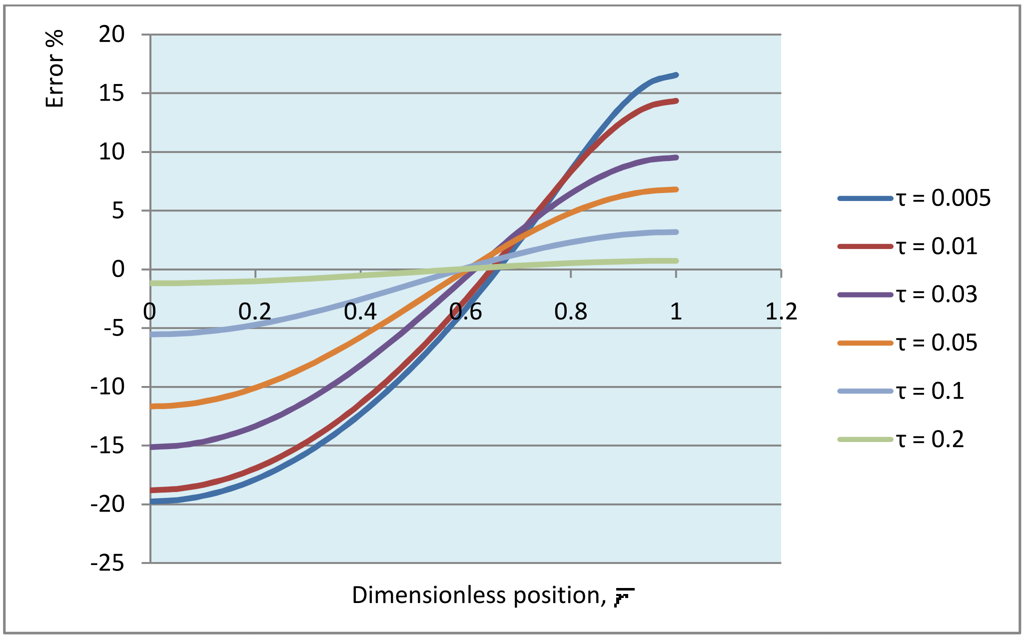

Figure 4.7: Variation of errors for long cylinder for Bi = 1

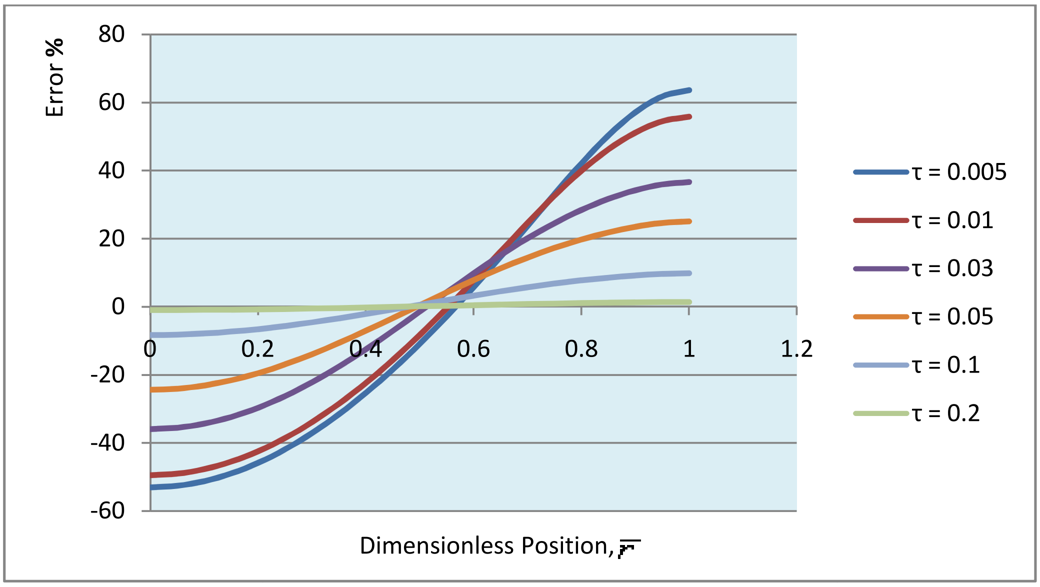

Figure 4.8: Variation of errors for long cylinder for Bi = 10

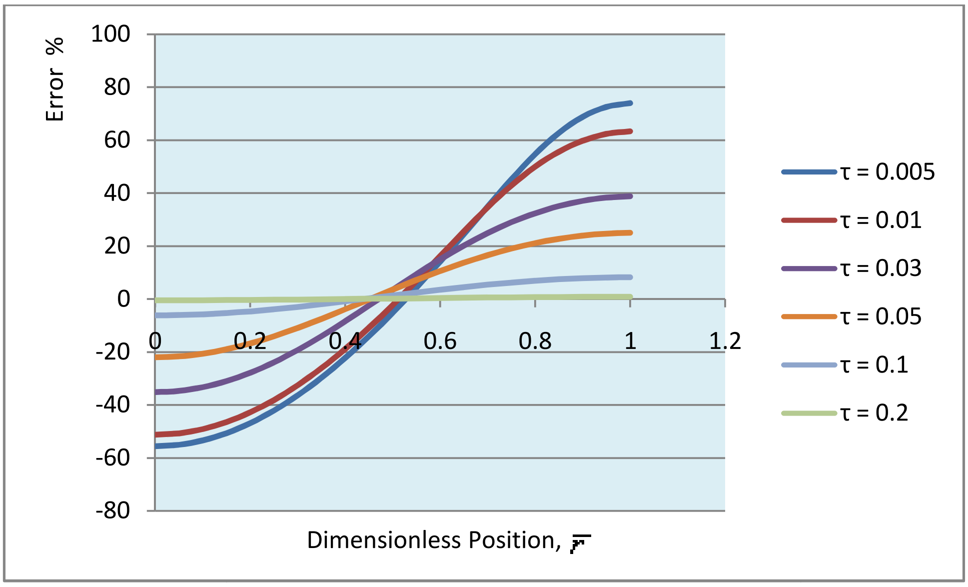

Figure 4.9: Variation of errors for long cylinder for Bi = 100

4.1.3 Analysis of the Error for Solid Sphere

Variation of errors figures have been obtained for Bi=1 and Bi=10 and Bi=100. For small dimensionless time, error is high. But for high dimensionless time, error is small. Also for small values of Biot number, error is small. But for high values of Biot number, error is high.

As can be seen Figure 4.10 and Figure 4.11, for example Bi=1; When it is τ = 0.005, local error is about 13%. When it is τ = 0.2, local error is about 1%. For example Bi=10; When it is τ = 0.005, error is about 60%. When it is τ = 0.2, local error is about 1%.

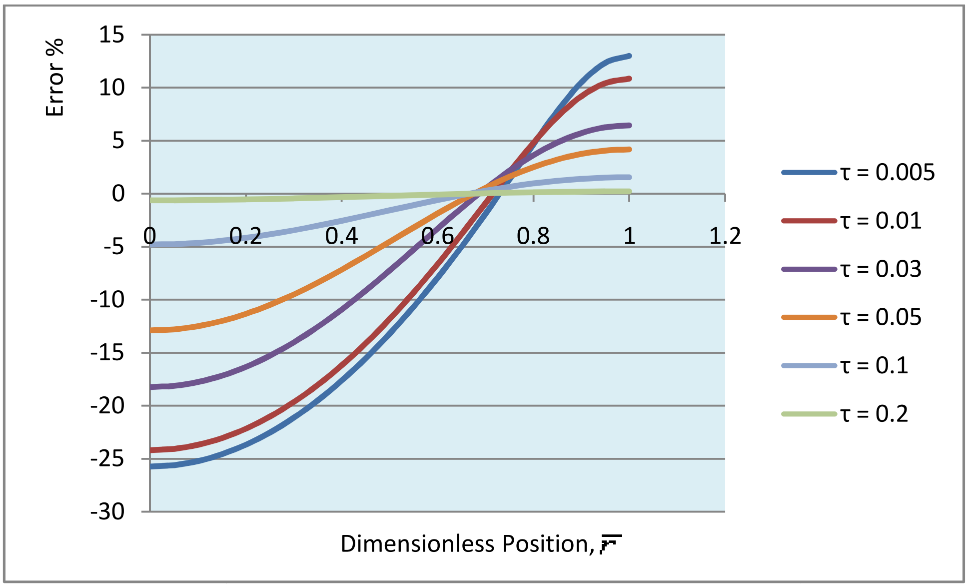

Figure 4.10: Variation of errors for solid sphere for Bi = 1

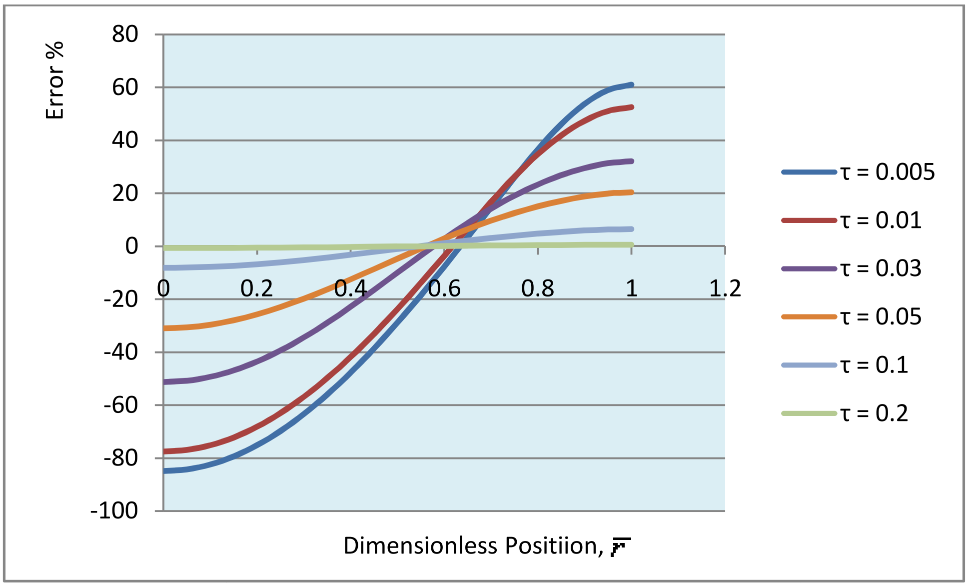

Figure 4.11: Variation of errors for solid sphere for Bi = 10

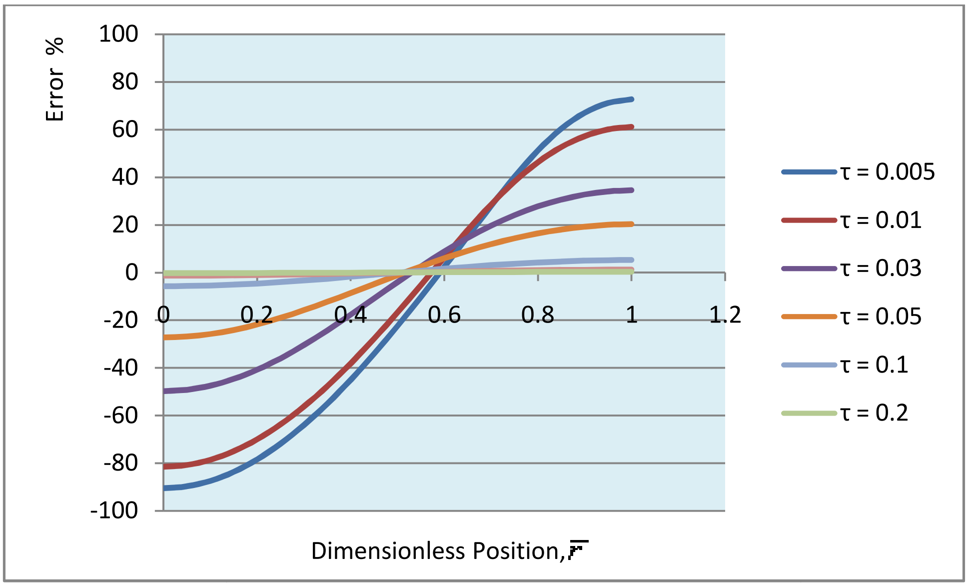

Figure 4.12: Variation of errors for solid sphere for Bi = 100

4.2 Correction Factor

One term approximation solution has been numerically compared with the exact solution. Exact solution has infinite series which are difficult to evaluate. Therefore, single correction factor that can be used with one term approximation method for dimensionless time less than 0.2 is defined between exact solution and one term approximation solution. This correction factor has been investigated for values of Biot number which are 1, 2, 3, 4, 5, 6, 7, 7, 8, 9, 10, 20, 30, 40, 50 and 100 and for values of dimensionless time which are 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.1 0.15, 0.2 and 0.25. Dimensionless positions for plane wall have been accepted with interval 0.1 from -1 to 1 and for long cylinder and solid sphere have been accepted with interval 0.05 from 0 to 1. Correction factor is defined as follows;

$$\theta_{exact} = \mathrm{C_f} * \theta_{one\ term} \tag{4.1}$$

Correction factor (C_f) is a function of dimensionless time, dimensionless position and Biot number. In this study for correction factor an equation with a simple form was looked for. But unique correction factor as a function of dimensionless time, dimensionless position and Biot number could not be obtained. Only for each Biot number, correction factors as a function of dimensionless time and dimensionless position have been obtained.

Correction factor which is a function of dimensionless time and dimensionless position is fourth degree polynomial function form. Below this function which can be used for three bodies has been shown. Fourth degree polynomial functions as follows;

$$\mathrm{C_f} = a + b * \bar{x} + c * \bar{x}^2 + d * \bar{x}^3 + e * \bar{x}^4 \tag{4.2}$$

where C_f is correction factor. $\bar{x}$ is dimensionless position. Coefficients of correction factor a, b, c, d and e are function of dimensionless time. Obtained this function is only for single Biot number. This fourth degree polynomial function can be used for wall, cylinder and sphere. But Coefficients of correction factor are different for each geometry. They have been shown in Table 4.1, Table 4.2 and Table 4.3.

Table 4.1: Coefficients of correction factor for plane wall

Bi=1	$a = 0.126329(7.982394 - e^{-8.415184\tau})$ b is negligible $c = (2.311062 - 10.056281 * \tau)/(1 + 310.95201 * \tau + 896.85645 * \tau^2)$ d is negligible $e = (0.008201 - 9.117113 * \tau)/(1 - 12.728689 * \tau + 896.85645 * \tau^2)$
Bi=2	$a = 0.184202(5.495517 - e^{-9.045679\tau})$ b is negligible $c = (0.650485 + 1.045287 * \tau)/(1 - 6.910827 * \tau + 202.16023 * \tau^2)$ d is negligible $e = (0.036905 - 9.880988 * \tau)/(1 - 23.709726 * \tau + 784.35183 * \tau^2)$
Bi=3	$a = 0.214799(4.722116 - e^{-9.567738\tau})$ b is negligible $c = (0.753273 + 3.211806 * \tau)/(1 - 9.864673 * \tau + 295.12798 * \tau^2)$ d is negligible $e = (-0.133650 - 1.232378 * \tau)/(1 - 19.153243 * \tau + 227.23079 * \tau^2)$
Bi=4	$a = 0.232671(4.364602 - e^{-9.975218\tau})$ b is negligible $c = (0.789106 + 6.357990 * \tau)/(1 - 13.05656 * \tau + 408.2901 * \tau^2)$ d is negligible $e = (-0.099003 - 2.254443 * \tau)/(1 - 22.737735 * \tau + 295.68679 * \tau^2)$
Bi=5	$a = 0.243864(4.167605 - e^{-10.289626\tau})$ b is negligible $c = (0.791346 + 9.667105 * \tau)/(1 - 16.109628 * \tau + 518.35543 * \tau^2)$ d is negligible $e = (-0.040149 - 4.258488 * \tau)/(1 - 27.257277 * \tau + 419.61342 * \tau^2)$
Bi=6	$a = 0.251244(4.047534 - e^{-10.52873\tau})$ b is negligible $c = (0.775123 + 13.086386 * \tau)/(1 - 18.914203 * \tau + 624.98074 * \tau^2)$ d is negligible $e = (0.003361 - 5.725279 * \tau)/(1 - 30.662849 * \tau + 510.28937 * \tau^2)$
Bi=7	$a = 0.256310(3.969263 - e^{-10.724676\tau})$ b is negligible $c = (0.748826 + 16.612947 * \tau)/(1 - 21.433594 * \tau + 728.56269 * \tau^2)$ d is negligible $e = (0.030774 - 6.615612 * \tau)/(1 - 33.021832 * \tau + 567.06309 * \tau^2)$

Bi=8	$a = 0.259908(3.915627 - e^{-10.879663\tau})$ b is negligible $c = (0.716623 + 20.351084 * \tau)/(1 - 23.663537 * \tau + 831.63007 * \tau^2)$ d is negligible $e = (0.048934 - 7.204529 * \tau)/(1 - 34.759714 * \tau + 606.24465 * \tau^2)$
Bi=9	$a = 0.262540(3.877352 - e^{-11.008465\tau})$ b is negligible $c = (0.681237 + 24.304514 * \tau)/(1 - 25.621247 * \tau + 934.5836 * \tau^2)$ d is negligible $e = (0.060725 - 7.597066 * \tau)/(1 - 36.060524 * \tau + 633.90329 * \tau^2)$
Bi=10	$a = 0.264519(3.849064 - e^{-11.118644\tau})$ b is negligible $c = (0.642898 + 28.660747 * \tau)/(1 - 27.343247 * \tau + 1041.841 * \tau^2)$ d is negligible $e = (0.068047 - 7.852905 * \tau)/(1 - 37.056681 * \tau + 653.78744 * \tau^2)$
Bi=20	$a = 0.272918(3.741004 - e^{-11.850315\tau})$ b is negligible $c = (0.232190 + 86.964776 * \tau)/(1 - 35.667439 * \tau + 225.4767 * \tau^2)$ d is negligible $e = (0.048985 - 7.762302 * \tau)/(1 - 39.396152 * \tau + 682.89178 * \tau^2)$
Bi=30	$a = 0.275135(3.696175 - e^{-12.385652\tau})$ b is negligible $c = (0.045256 + 120.36849 * \tau)/(1 - 38.403729 * \tau + 2918.2652 * \tau^2)$ d is negligible $e = (0.005199 - 6.912352 * \tau)/(1 - 38.505398 * \tau + 652.66157 * \tau^2)$
Bi=40	$a = 0.277365(3.662810 - e^{-12.791046\tau})$ b is negligible $c = (0.028725 + 0.980664 * \tau)/(1 - 1.301427 * \tau + 625.25763 * \tau^2)$ d is negligible $e = (-0.029586 - 6.241722 * \tau)/(1 - 37.453172 * \tau + 625.25763 * \tau^2)$
Bi=50	$a = 0.279108(3.636612 - e^{-13.128096\tau})$ b is negligible $c = (-0.004022 + 137.72874 * \tau)/(1 - 40.577794 * \tau + 3284.5959 * \tau^2)$ d is negligible $e = (-0.055696 - 5.752320 * \tau)/(1 - 36.575441 * \tau + 604.40141 * \tau^2)$

Bi=100	$a = 0.284155(3.563424 - e^{-14.046618\tau})$ b is negligible $c = (0.029146 + 0.981895 * \tau)/(1 - 1.291975 * \tau + 549.23933 * \tau^2)$ d is negligible $e = (-0.123664 - 4.521896 * \tau)/(1 - 34.055298 * \tau + 549.23933 * \tau^2)$

Table 4.2: Coefficients of correction factor for long cylinder

Bi=1	$a = 0.822081(1.637523 - e^{-2.849864\tau})$ $b = (0.151391 - 2.860969 * \tau)/(1 - 39.876411 * \tau + 273.5311 * \tau^2)$ $c = (-0.656625 + 25.478695 * \tau)/(1 - 40.474817 * \tau + 1283.8519 * \tau^2)$ $d = 1/(1.777145 - 26.410556 * \tau^{-34.688904})$ $e = (-0.928126 + 1.266053 * \tau)/(1 - 32.050653 * \tau + 161.8129 * \tau^2)$
Bi=2	$a = 0.288628(3.519442 - e^{-11.074542\tau})$ $b = (0.304413 - 3.927650 * \tau)/(1 - 38.703681 * \tau + 2885.1187 * \tau^2$ $c = (-1.430896 + 46.092216 * \tau)/(1 - 43.869029 * \tau + 1595.516 * \tau^2)$ $d = 1/(3.378644 - 34.08594 * \tau^{-36.206308})$ $e = (-1.602017 - 5.427165 * \tau)/(1 - 36.950612 * \tau + 1888.5007 * \tau^2)$
Bi=3	$a = 0.340323(2.990044 - e^{-11.609067\tau})$ $b = (0.436719 - 4.562605 * \tau)/(1 - 40.521028 * \tau + 3007.8859 * \tau^2)$ $c = (-2.098973 + 60.607715 * \tau)/(1 - 47.284703 * \tau + 1826.5988 * \tau^2)$ $d = 1/(4.571668 - 35.205391 * \tau^{-40.289489})$ $e = (-1.966843 - 17.756532 * \tau)/(1 - 44.809442 * \tau + 2215.8997 * \tau^2)$
Bi=4	$a = 0.371094(2.743006 - e^{-12.139356\tau})$ $b = (0.541335 - 5.204513 * \tau)/(1 + 43.803591 * \tau + 3134.0487 * \tau^2)$ $c = (-2.616986 + 71.201932 * \tau)/(1 - 51.005834 * \tau + 2010.7802 * \tau^2)$ $d = 1/(0.152850 + 17475.246 * \tau^{3.2364331})$ $e = (-2.062545 - 36.813728 * \tau)/(1 - 54.424316 * \tau + 2669.0563 * \tau^2)$
Bi=5	$a = 0.390820(2.604233 - e^{-12.630452\tau})$ $b = (0.621213 - 5.895231 * \tau)/(1 - 47.614967 * \tau + 3262.3359 * \tau^2$ $c = (-2.998884 + 79.31566 * \tau)/(1 - 54.876927 * \tau + 2168.1578 * \tau^2)$ $d = 1/(0.133953 + 218.17 * \tau^{3.338775})$ $e = (-1.981229 - 59.86601 * \tau)/(1 - 64.316893 * \tau + 3162.8025 * \tau^2)$
Bi=6	$a = 0.404243(2.516187 - e^{-13.068763\tau})$ $b = (0.681409 - 6.605119 * \tau)/(1 - 51.45904 * \tau + 3386.3524 * \tau^2$ $c = (-3.273622 + 85.719795 * \tau)/(1 - 58.700261 * \tau + 2307.6985 * \tau^2)$ $d = 1/(0.122379 + 2956.715 * \tau^{3.449253})$ $e = (-1.815786 - 82.400673 * \tau)/(1 - 73.396596 * \tau + 3613.8638 * \tau^2)$

Bi=7	$a = 0.413816(2.457312 - e^{-13.455651\tau})$ $b = (0.726937 - 7.305665 * \tau)/(1 - 55.095419 * \tau + 3502.34 * \tau^2$ $c = (-3.469638 + 90.880444 * \tau)/(1 - 62.340654 * \tau + 2433.558 * \tau^2)$ $d = 1/(0.114678 + 411.597999 * \tau^{3.557869})$ $e = (-1.62142 - 102.28486 * \tau)/(1 - 81.372477 * \tau + 3999.11 * \tau^2)$
Bi=8	$a = 0.420909(2.414375 - e^{-13.796484\tau})$ $b = (0.761910 - 7.979449 * \tau)/(1 - 58.422232 * \tau - 58.422232 * \tau^2)$ $c = (-3.610388 + 95.130101 * \tau)/(1 - 65.712219 * \tau + 2547.3766 * \tau^2)$ $d = 1/(0.109081 + 4769.58 * \tau^{3.605247})$ $e = (-1.427256 - 118.9667 * \tau)/(1 - 88.247172 * \tau + 439.6239 * \tau^2)$
Bi=9	$a = 0.426332(2.382929 - e^{-14.097501\tau})$ $b = (0.789543 - 8.625721 * \tau)/(1 - 61.391974 * \tau + 3705.4418 * \tau^2)$ $c = (-3.713747 + 98.724285 * \tau)/(1 - 68.767186 * \tau + 2649.8059 * \tau^2)$ $d = 1/(0.104934 + 6046.029 * \tau^{3.682979})$ $e = (-1.246655 - 132.67805 * \tau)/(1 - 94.137833 * \tau + 4585.2033 * \tau^2)$
Bi=10	$a = 0.430589(2.358364 - e^{-14.364608\tau})$ $b = (0.812122 - 9.242884 * \tau)/(1 - 64.008911 * \tau + 3792.2372 * \tau^2)$ $c = (-3.792828 + 101.86092 * \tau)/(1 - 71.485632 * \tau + 2741.2333 * \tau^2)$ $d = 1/(-721.5221 - 5.628772 * \tau^{-37.698008})$ $e = (-37.698008 - 143.76191 * \tau)/(1 - 99.157261 * \tau + 4804.1957 * \tau^2)$
Bi=20	$a = 0.448749(2.256597 - e^{-15.979349\tau})$ $b = (0.959026 - 14.370742 * \tau)/(1 - 76.509816 * \tau + 4322.0891 * \tau^2)$ $c = (-4.326157 + 126.59451 * \tau)/(1 - 84.025541 * \tau + 3210.7997 * \tau^2)$ $d = 1/(0.085018 + 161.886 * \tau^{3.690292})$ $e = (-0.454262 - 178.04524 * \tau)/(1 - 121.46485 * \tau + 5657.0242 * \tau^2)$
Bi=30	$a = 0.454987(2.222398 - e^{-16.772306\tau})$ $b = (0.679596 + 89781.48 * \tau)/(1 + 3.095734 * \tau + 0.435983 * \tau^2)$ $c = (-4.936574 + 149.81835 * \tau)/(1 - 84.691913 * \tau + 3340.424 * \tau^2)$ $d = 1/(0.076550 + 69783.304 * \tau^{3.690958})$ $e = (-0.620604 - 168.64611 * \tau)/(1 - 124.41622 * \tau + 5699.6652 * \tau^2)$
Bi=40	$a = 0.458530(2.203209 - e^{-17.259837\tau})$ $b = (0.629189 + 2072.94 * \tau)/(1 + 3.106842 * \tau - 5.527244 * \tau^2)$ $c = (-5.527744 + 170.2087 * \tau)/(1 - 82.937558 * \tau + 3405.8695 * \tau^2)$ $d = 1/(-44.957316 - 238 * \tau^{-46.064148})$ $e = (-0.950312 - 154.07565 * \tau)/(1 - 123.09049 * \tau + 505.3206 * \tau^2)$

Bi=50	$a = 0.460910(2.190465 - e^{-17.592338\tau})$ $b = (0.589572 + 13195.72 * \tau)/(1 + 2.980723 * \tau + 2.194657 * \tau^2)$ $c = (-6.038597 + 187.12899 * \tau)/(1 + 80.951293 * \tau + 3459.2234 * \tau^2)$ $d = 0.734944(5.1604657 - e^{-13.592311})$ $e = (-0.454262 - 178.04524 * \tau)/(1 - 121.46485 * \tau + 5657.0242 * \tau^2)$
Bi=100	$a = 0.430589(2.358364 - e^{-14.364608\tau})$ $b = (0.959026 - 14.370742 * \tau)/(1 - 76.509816 * \tau + 4322.0891 * \tau^2)$ $c = (-4.326157 + 126.59451 * \tau)/(1 - 84.025541 * \tau + 3210.7997 * \tau^2)$ $d = 0.114944(1.242346 - e^{-10.594545\tau})$ $e = (-0.454625 - 178.04524 * \tau)/(1 - 121.46485 * \tau + 5657.0242 * \tau^2)$

Table 4.3: Coefficients of correction factor for solid sphere

Bi=1	$a = 0.240473(4.203283 - e^{-13.363307\tau})$ $b = (0.146156 - 3.587276 * \tau)/(1 - 50.225324 * \tau + 315.3488 * \tau^2)$ $c = (-0.616436 + 27.040998 * \tau)/(1 - 51.722185 * \tau + 16.518 * \tau^2)$ $d = (1.707603 - 32.959621 * \tau)/(1 - 45.143975 * \tau + 2100.5739 * \tau^2)$ $e = (-0.908747 + 4.971762 * \tau)/(1 - 40.407269 * \tau + 2029.5616 * \tau^2)$
Bi=2	$a = 0.367824(2.765256 - e^{-13.320272\tau})$ $b = (0.293578 - 5.103238 * \tau)/(1 - 47.159809 * \tau + 320.1102 * \tau^2)$ $c = (-1.356186 + 49.648762 * \tau)/(1 - 53.620613 * \tau + 199.6841 * \tau^2)$ $d = (3.253365 - 44.049696 * \tau)/(1 - 44.494819 * \tau + 219.911 * \tau^2)$ $e = (-1.578448 + 0.875843 * \tau)/(1 - 43.311784 * \tau + 213.4825 * \tau^2)$
Bi=3	$a = 0.439921(2.319381 - e^{-13.556686\tau})$ $b = (0.422265 - 5.893876 * \tau)/(1 - 47.336382 * \tau + 345.086 * \tau^2)$ $c = (-2.011424 + 65.274401 * \tau)/(1 - 55.515745 * \tau + 121.0353 * \tau^2)$ $d = (4.431950 - 46.748029 * \tau)/(1 - 46.959047 * \tau + 230.982 * \tau^2)$ $e = (-1.964060 - 8.424815 * \tau)/(1 - 49.635468 * \tau + 232.3827 * \tau^2)$
Bi=4	$a = 0.483584(2.112551 - e^{-13.904238\tau})$ $b = (0.526428 - 6.571484 * \tau)/(1 - 49.416188 * \tau + 3424.6186 * \tau^2)$ $c = (-2.532487 + 76.34942 * \tau)/(1 - 58.003243 * \tau + 2280.8224 * \tau^2)$ $d = (7.056811 + 0.000599 * \tau)/(1 - 0.355396 * \tau + 1.987744 * \tau^2)$ $e = (-2.100706 - 22.835746 * \tau)/(1 - 57.97754 * \tau + 2743.3224 * \tau^2)$
Bi=5	$a = 0.511702(1.997660 - e^{-14.279418\tau})$ $b = (0.607373 - 7.269025 * \tau)/(1 - 52.397576 * \tau + 3517.7519 * \tau^2)$ $c = (-2.924460 + 84.709019 * \tau)/(1 - 60.917545 * \tau + 2413.5178 * \tau^2)$ $d = (8.204788 + 0.000984 * \tau)/(1 - 0.479029 * \tau + 1.852629 * \tau^2)$ $e = (-2.063998 - 40.720739 * \tau)/(1 - 66.967327 * \tau + 3157.2638 * \tau^2)$

Bi = 6	$a = 0.530241(1.925528 - e^{-14.644471\tau})$ $b = (0.669696 - 7.988845 * \tau)/(1 - 55.677635 * \tau + 3615.888 * \tau^2)$ $c = (-3.212123 + 91.309734 * \tau)/(1 - 63.997724 * \tau + 2530.0326 * \tau^2)$ $d = (0.122670 + 50242.585 * \tau)/(1 + 3.540981 * \tau + 4.365229 * \tau^2)$ $e = (-1.931337 - 59.126359 * \tau)/(1 - 75.55919 * \tau + 3557.8979 * \tau^2)$
Bi = 7	$a = 0.544282(1.877596 - e^{-14.984259\tau})$ $b = (0.716285 - 8.704378 * \tau)/(1 - 58.930033 * \tau + 3712.8763 * \tau^2)$ $c = (-3.420965 + 96.676095 * \tau)/(1 - 67.042695 * \tau + 2634.5785 * \tau^2)$ $d = (0.114577 + 55143.639 * \tau)/(1 + 3.588389 * \tau + 0.114577 * \tau^2)$ $e = (-1.759091 - 75.948381 * \tau)/(1 - 83.271087 * \tau + 3911.8339 * \tau^2)$
Bi = 8	$a = 0.554208(1.843343 - e^{-15.29428\tau})$ $b = (0.752638 - 9.398989 * \tau)/(1 - 61.987101 * \tau + 3805.2126 * \tau^2)$ $c = (-3.573241 + 101.14321 * \tau)/(1 - 69.929059 * \tau + 2729.0937 * \tau^2)$ $d = (0.108896 + 6381.552 * \tau)/(1 + 3.644276 * \tau + 7.988845 * \tau^2)$ $e = (-1.578291 - 90.473415 * \tau)/(1 - 90.022288 * \tau + 4214.2014 * \tau^2)$
Bi = 9	$a = 0.561728(1.817985 - e^{-15.574803\tau})$ $b = (0.781301 - 10.06557 * \tau)/(1 - 64.776892 * \tau + 3891.1949 * \tau^2)$ $c = (-3.686411 + 104.95483 * \tau)/(1 - 72.585884 * \tau + 2814.4015 * \tau^2)$ $d = (0.104641 + 6997.71 * \tau)/(1 + 3.682422 * \tau + 1.664552 * \tau^2)$ $e = (-0.908745 + 4.9717622 * \tau)/(1 - 40.407269 * \tau + 2029.5616 * \tau^2)$
Bi = 10	$a = 0.567384(1.798940 - e^{-15.848819\tau})$ $b = (0.793508 - 11.602121 * \tau)/(1 - 70.495801 * \tau + 4052.7889 * \tau^2)$ $c = (-3.706028 + 110.82921 * \tau)/(1 - 77.816196 * \tau + 248.0408 * \tau^2)$ $d = 0.100444(1 + 4242.673 * \tau + 3.526408 * \tau^2)$ $e = (-1.267816 - 101.57453 * \tau)/(1 - 102.73876 * \tau + 4678.4185 * \tau^2)$
Bi = 20	$a = 0.591820(1.719766 - e^{-17.47152\tau})$ $b = (-1.912665 + 0.022460 * \tau)/(1 + 1.665016 * \tau^2)$ $c = (-4.457625 + 142.14042 * \tau)/(1 - 83.094515 * \tau + 3085.0963 * \tau^2)$ $d = 0.567384/(1.798940 - e^{-15.848819\tau})$ $e = (-0.726469 - 119.38872 * \tau)/(1 - 122.79877 * \tau + 5299.7524 * \tau^2)$
Bi = 30	$a = 0.613512(1.654015 - e^{-18.79427\tau})$ $b = -52.53(1 - 19976.64 * \tau - 62.186113 * \tau^2)$ $c = (-3.501868 + 121.53256 * \tau)/(1 - 135.68102 * \tau + 5240.7784 * \tau^2)$ $d = (0.174831 - 2.374422 * \tau)/(1 - 1.64327 * \tau + 65.053422 * \tau^2)$ $e = (0.169611 - 242.3722 * \tau)/(1 - 166.29657 * \tau + 821.039199 * \tau^2)$

Bi = 40	$a = 0.603263(1.683189 - e^{-18.656879\tau})$ $b = -2.369793(1 + 0.015991 * \tau + 69.456121 * \tau^2)$ $c = (-5.514491 + 178.4064 * \tau)/(1 - 85.698252 * \tau + 3531.0218 * \tau^2)$ $d = 0.511762 + 0.786142/\tau$ $e = (-1.079334 - 120.59044 * \tau)/(1 - 124.46657 * \tau + 5472.0209 * \tau^2)$
Bi = 50	$a = 0.554208(1.843343 - e^{-15.29428\tau})$ $b = (0.752638 - 9.398989 * \tau)/(1 - 61.987101 * \tau + 3805.2126 * \tau^2)$ $c = (-3.573241 + 101.14321 * \tau)/(1 - 69.929059 * \tau + 2729.0937 * \tau^2)$ $d = (0.108896 + 63814.552 * \tau)/(1 + 3.644763 * \tau + 7.988459 * \tau^2)$ $e = (-1.578291 - 90.473415 * \tau)/(1 - 90.022288 * \tau + 4214.2014 * \tau^2)$
Bi = 100	$a = 0.561728(1.817985 - e^{-15.574803\tau})$ $b = (0.781301 - 10.06557 * \tau)/(1 - 64.776892 * \tau + 3891.1949 * \tau^2)$ $c = (-3.686411 + 104.95483 * \tau)/(1 - 72.585884 * \tau + 2814.4015 * \tau^2)$ $d = (0.104641 + 6991.71 * \tau)/(1 + 3.680422 * \tau + 1.663552 * \tau^2)$ $e = (-0.908745 + 4.971622 * \tau)/(1 - 40.407269 * \tau + 2029.5616 * \tau^2)$

Variation of correction factor can be seen for plane wall from Figure 4.13, Figure 4.14 and Figure 4.15, for long cylinder from Figure 4.16, Figure 4.17 and Figure 4.18 and for solid sphere from Figure 4.19, Figure 4.20 and Figure 4.21. As can be seen from figures, effect of dimensionless time, dimensionless position and Biot number on correction factor have been investigated. When dimensionless time increases, value of the correction factor decreases. When Biot number increases, value of the correction factor increases too.

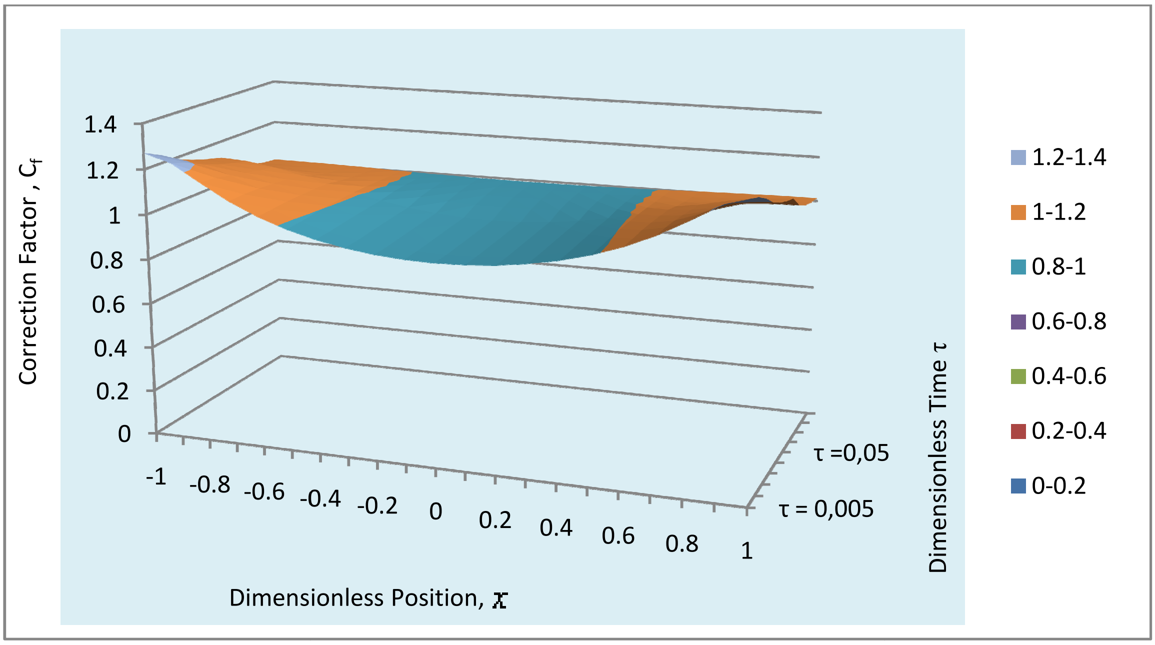

Figure 4.13: Variation of correction factor for plane wall for Bi = 1

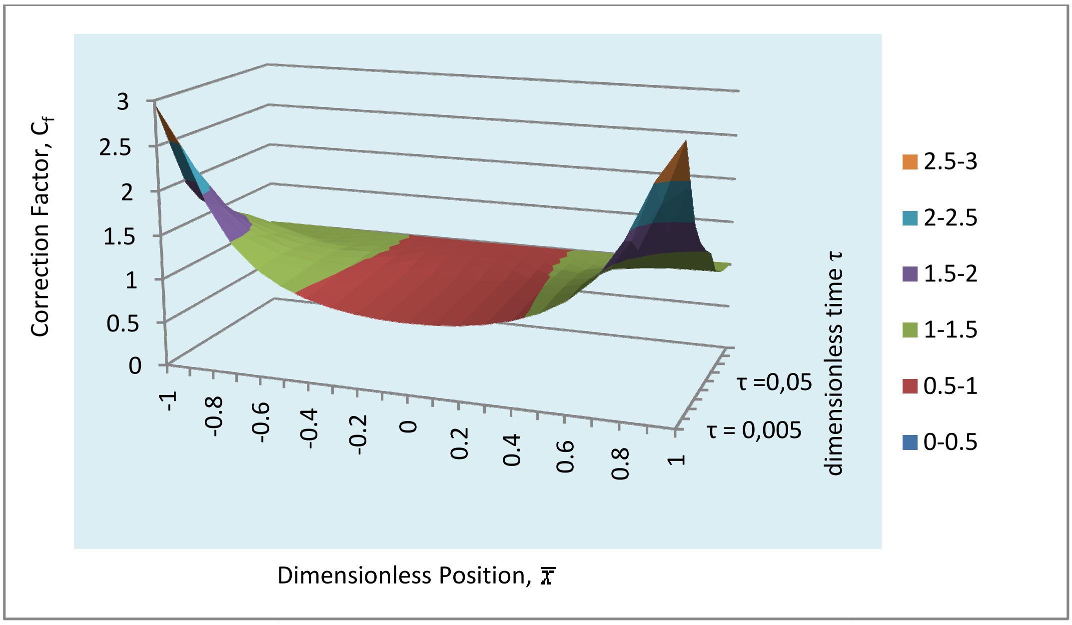

Figure 4.14: Variation of correction factor for plane wall Bi = 10

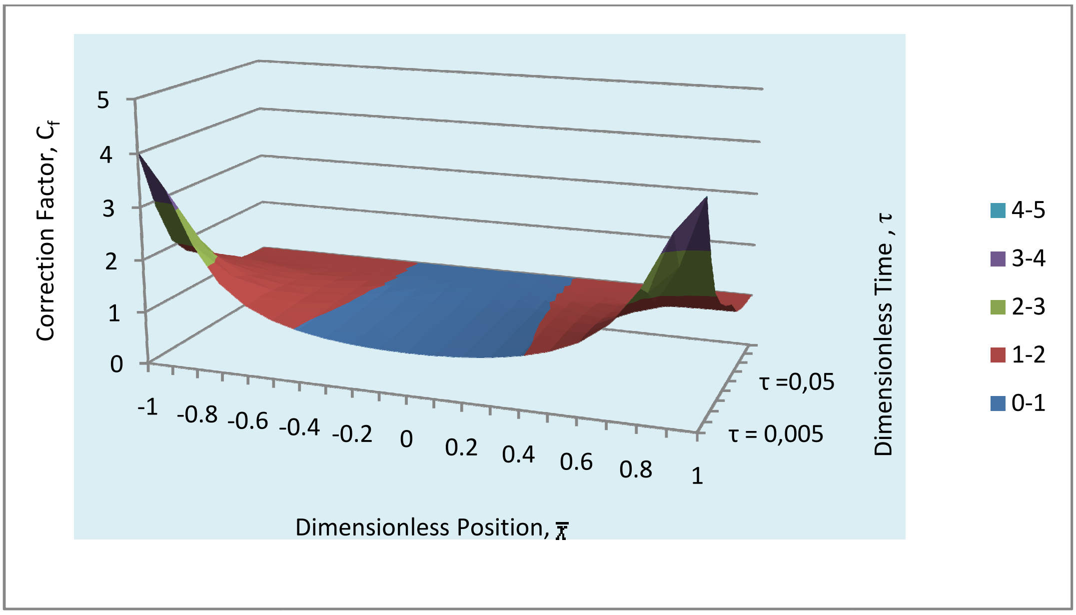

Figure 4.15: Variation of correction factor for plane wall Bi = 100

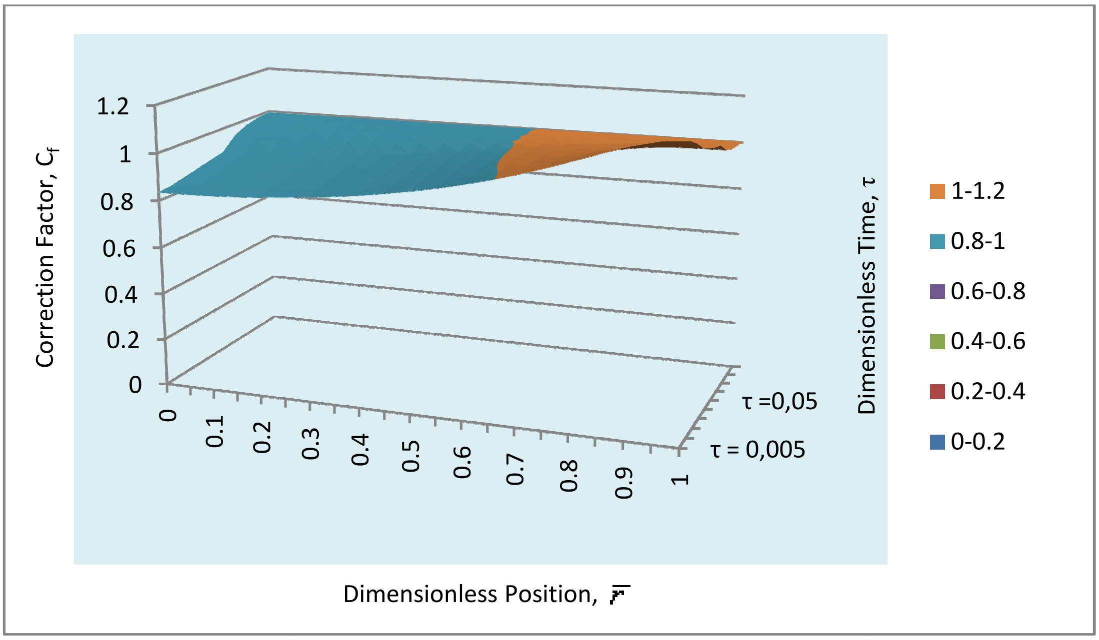

Figure 4.16: Variation of correction factor for long cylinder Bi = 1

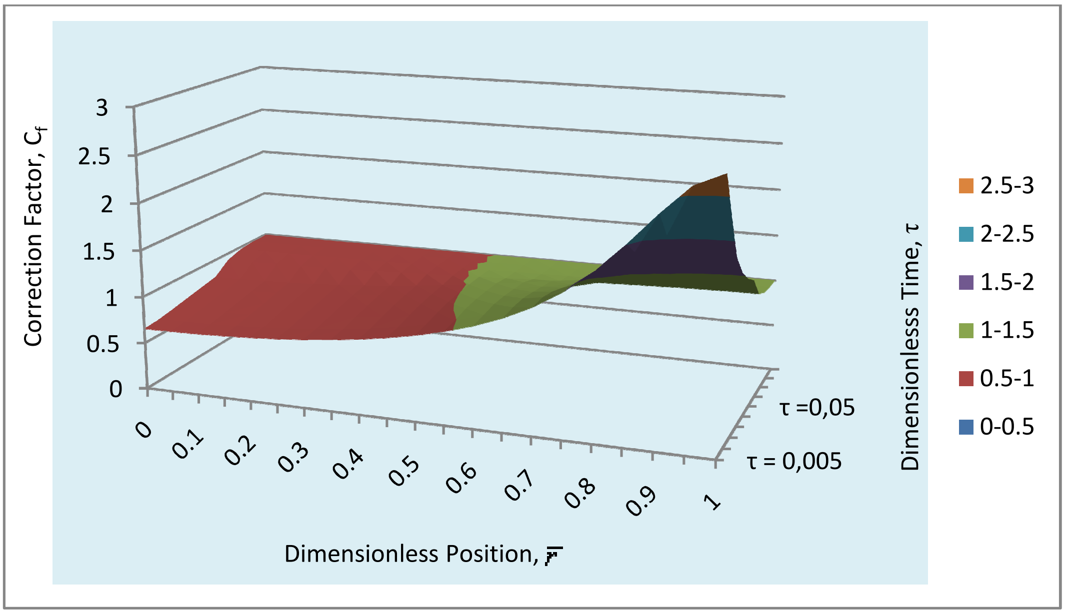

Figure 4.17: Variation of correction factor for long cylinder Bi = 10

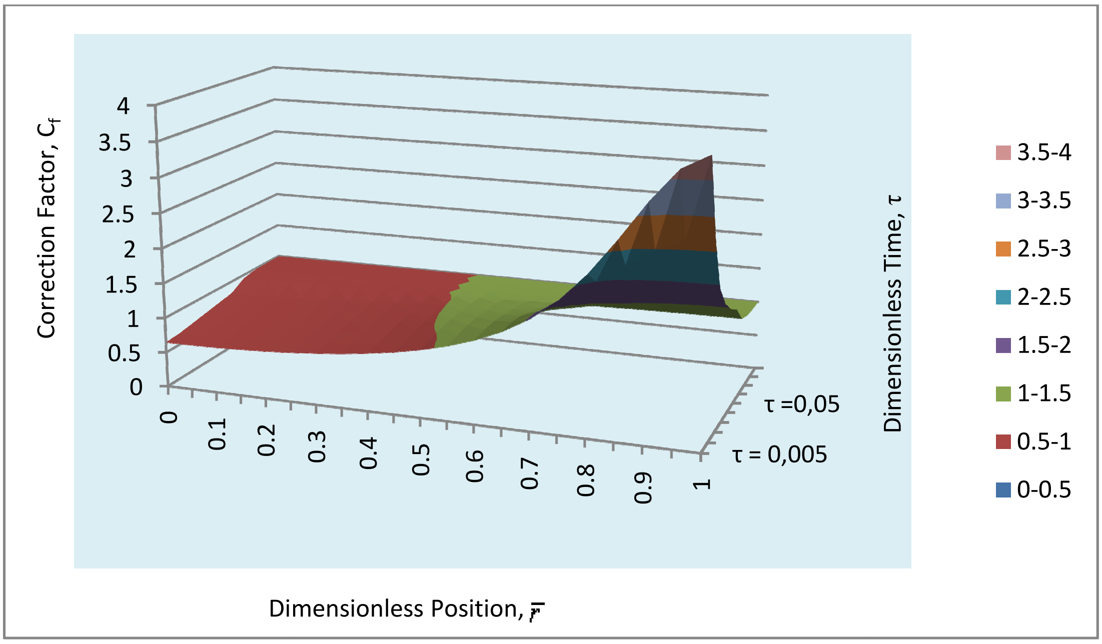

Figure 4.18: Variation of correction factor for long cylinder Bi = 100

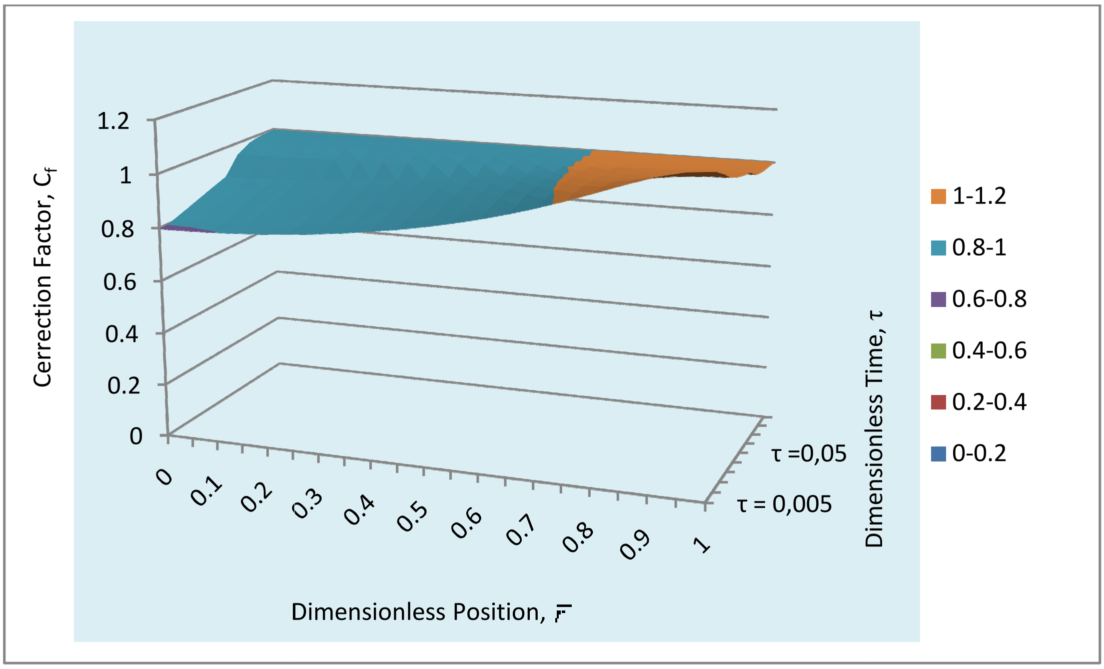

Figure 4.19: Variation of correction factor for sphere Bi = 1

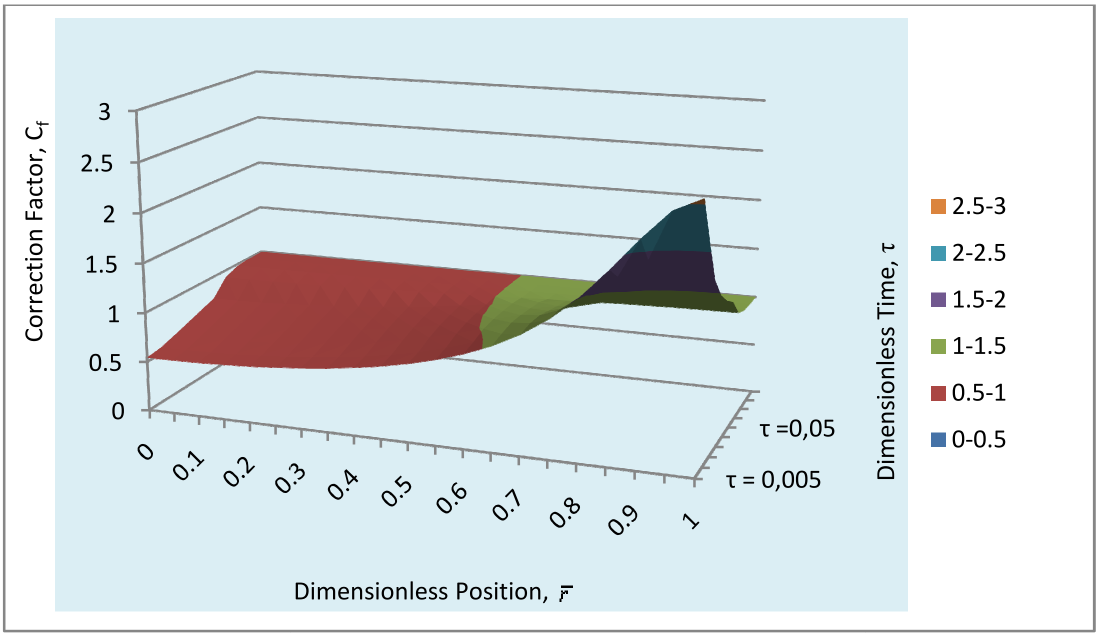

Figure 4.20: Variation of correction factor for sphere Bi = 10

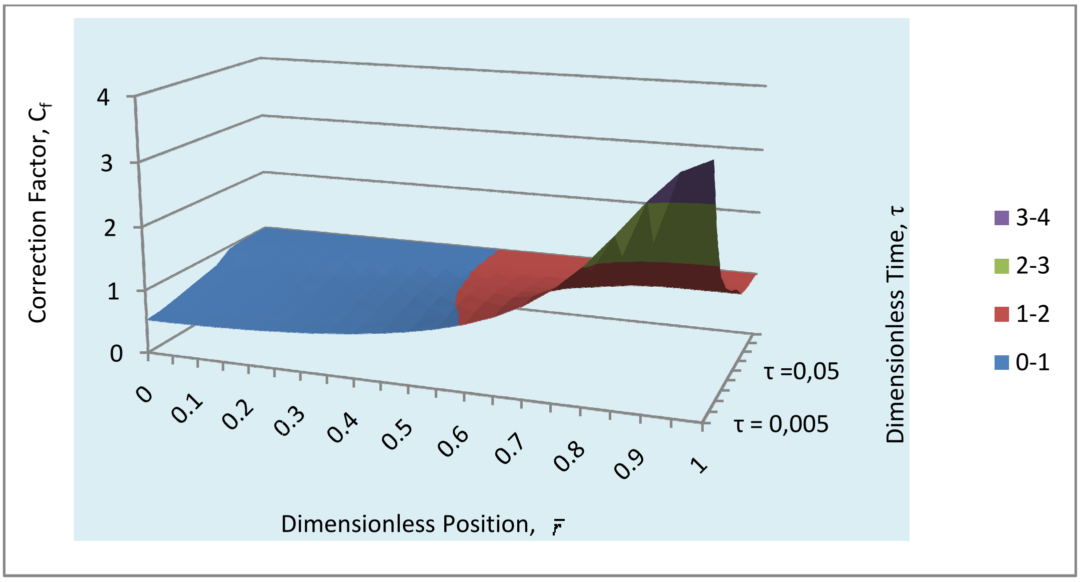

Figure 4.21: Variation of correction factor for sphere Bi = 100

4.3 Two Term Approximation Solution

The analytical solution was shown for one dimensional transient heat conduction involves infinite series which are difficult to evaluate. Therefore, to minimize error in one term approximation solution has been considered to take two terms in the exact solution. This solution has been called two term approximation method. Two term approximation solution has been defined as follows;

Plane wall $$\theta_{wall} = A_1 \cos(\lambda_1 \bar{x})\, e^{-\lambda_1{}^2 \tau} + A_2 \cos(\lambda_2 \bar{x})\, e^{-\lambda_2{}^2 \tau} \quad (4.19)$$

Cylinder $$\theta_{cyl.} = A_1 J_0(\lambda_1 \bar{r}) e^{-\lambda_1{}^2 \tau} + A_2 J_0(\lambda_2 \bar{r}) e^{-\lambda_2{}^2 \tau} \quad (4.20)$$

Sphere $$\theta_{sph.} = A_1 \frac{\sin(\lambda_1 \bar{r})}{(\lambda_1 \bar{r})} e^{-\lambda_1{}^2 \tau} + A_2 \frac{\sin(\lambda_2 \bar{r})}{(\lambda_2 \bar{r})} e^{-\lambda_2{}^2 \tau} \quad (4.21)$$

where the constants A_1 and A_2 for three geometries are expressed in Table 4.4. The constants A_1, A_2, λ_1 and λ_2 are functions of the Biot number only. Their values were listed in Table 4.3, Table 4.4 and Table 4.5 against the Biot number for three geometries.

Table 4.4: Coefficients used in the two term approximation solution

Coefficients W	Plane Wall	Long Cylinder	Solid Shpere
A_1	$\frac{4 \sin \lambda_1}{2\lambda_1 + \sin(2\lambda_1)}$	$\frac{2}{\lambda_1} \frac{J_1(\lambda_1)}{J^2{}_0(\lambda_1) + J^2{}_1(\lambda_1)}$	$\frac{4(\sin \lambda_1 - \lambda_1 \cos \lambda_1)}{2\lambda_1 - \sin(\lambda_1)}$
A_2	$\frac{4 \sin \lambda_2}{2\lambda_2 + \sin(2\lambda_2)}$	$\frac{2}{\lambda_2} \frac{J_1(\lambda_2)}{J^2{}_0(\lambda_2) + J^2{}_1(\lambda_2)}$	$\frac{4(\sin \lambda_2 - \lambda_1 \cos \lambda_2)}{2\lambda_2 - \sin(\lambda_2)}$

Where the roots λ_1 and λ_2 for three geometry as;

For plane wall, λ_1 and λ_2 are first and second roots of $Bi = \lambda_n \tan \lambda_n$

For long cylinder, λ_1 and λ_2 are first and second roots of $Bi = \lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)}$

For solid sphere, λ_1 and λ_2 are first and second roots of $Bi = 1 - \lambda_n \cos \lambda_n$

Table 4.5: Values of Coefficients used in the two term approximation solution for plane wall

Bi	λ_1	A_1	λ_2	A_2
1	0.8603	1.1191	3.4256	-0.1516
2	1.0768	1.1784	3.6435	-0.2367
3	1.1924	1.2102	3.8087	-0.2881
4	1.2645	1.2287	3.9351	-0.3214
5	1.3138	1.2402	4.0335	-0.3442
6	1.3495	1.2478	4.1116	-0.3603
7	1.3766	1.2531	4.1746	-0.3722
8	1.3978	1.2569	4.2263	-0.3811
9	1.4148	1.2598	4.2694	-0.3880
10	1.4288	1.2619	4.3058	-0.3934
20	1.4961	1.2699	4.4914	-0.4147
30	1.5201	1.2716	4.5614	-0.4197
40	1.5325	1.2723	4.5979	-0.4217
50	1.5400	1.2726	4.6202	-0.4226
100	1.5552	1.2730	4.6657	-0.4239

Table 4.6: Values of coefficients used in the two term approximation solution for long cylinder

Bi	λ_1	A_1	λ_2	A_2
1	1.2557	1.2070	4.0794	-0.2901
2	1.2557	1.2070	4.0794	-0.2901
3	1.7886	1.4190	4.4633	-0.6309
4	1.9080	1.4697	4.6018	-0.7278
5	1.9898	1.5028	4.7131	-0.7973
6	2.0490	1.5253	4.8033	-0.8484
7	2.0937	1.5411	4.8771	-0.8868
8	2.1286	1.5525	4.9383	-0.9163
9	2.1566	1.5611	4.9897	-0.9392
10	2.1794	1.5676	5.0332	-0.9575
20	2.2880	1.5919	5.2568	-1.0309
30	2.3261	1.5972	5.3409	-1.0486
40	2.3455	1.5992	5.3846	-1.0554
50	2.3572	1.6002	5.4111	-1.0587
100	2.3809	1.6015	5.4652	-1.0632

Table 4.7: Values of Coefficients used in the two term approximation solution for solid Sphere

Bi	λ_1	A_1	λ_2	A_2
1	1.5707	1.2732	4.7123	-0.4244
2	2.0287	1.4793	4.9131	-0.7672
3	2.2889	1.6226	5.0869	-1.0288
4	2.4556	1.7201	5.2329	-1.2252
5	2.5704	1.7870	5.3540	-1.3733
6	2.6536	1.8337	5.4543	-1.4860
7	2.7164	1.8673	5.5378	-1.5730
8	2.7653	1.8920	5.6077	-1.6410
9	2.8044	1.9106	5.6668	-1.6949
10	2.8363	1.9249	5.7172	-1.7381
20	2.9857	1.9781	5.9783	-1.9164
30	3.0372	1.9898	6.0766	-1.9602
40	3.0632	1.9941	6.1273	-1.9769
50	3.0788	1.9962	6.1581	-1.9850
100	3.1101	1.9990	6.2204	-1.9961

Values of two term approximation are more close to the values of the exact solution. It can be seen for plane wall, long cylinder and solid sphere from Figure 4.22, Figure 4.23 and Figure 4.24.

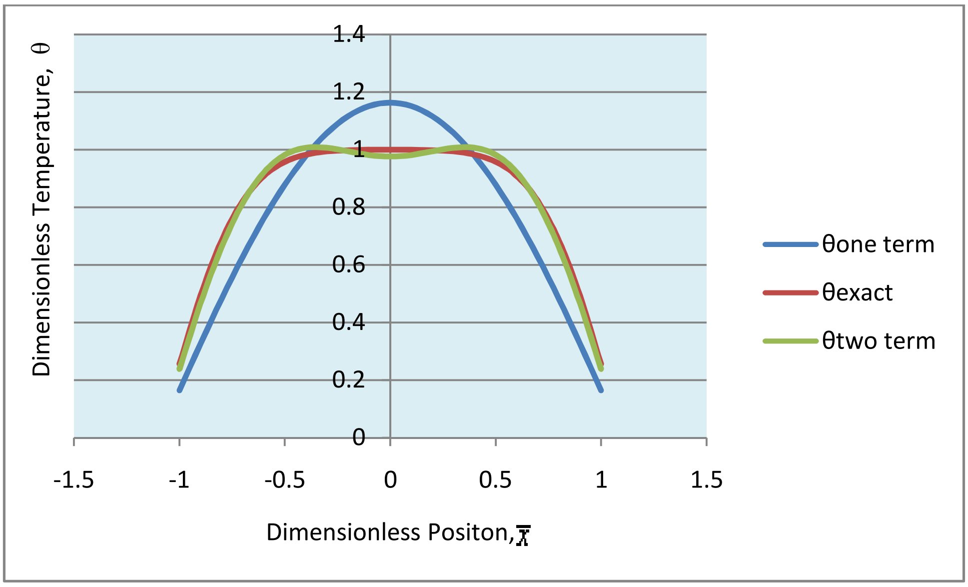

Figure 4.22: Difference between exact solution and approach solutions for plane wall for Bi = 10 and $\tau = 0.05$

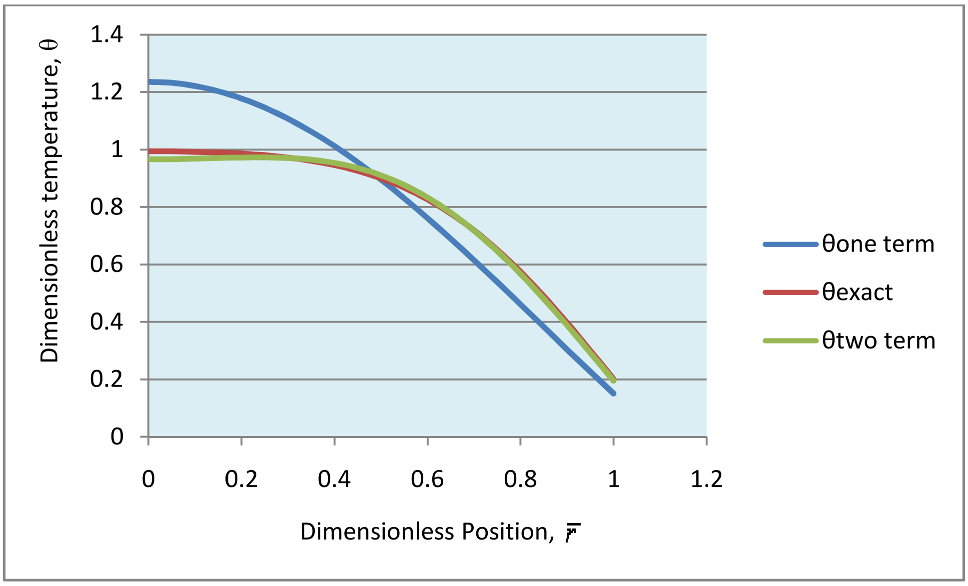

Figure 4.23: Difference between exact solution and approach solution for long cylinder for Bi = 10 and $\tau = 0.05$

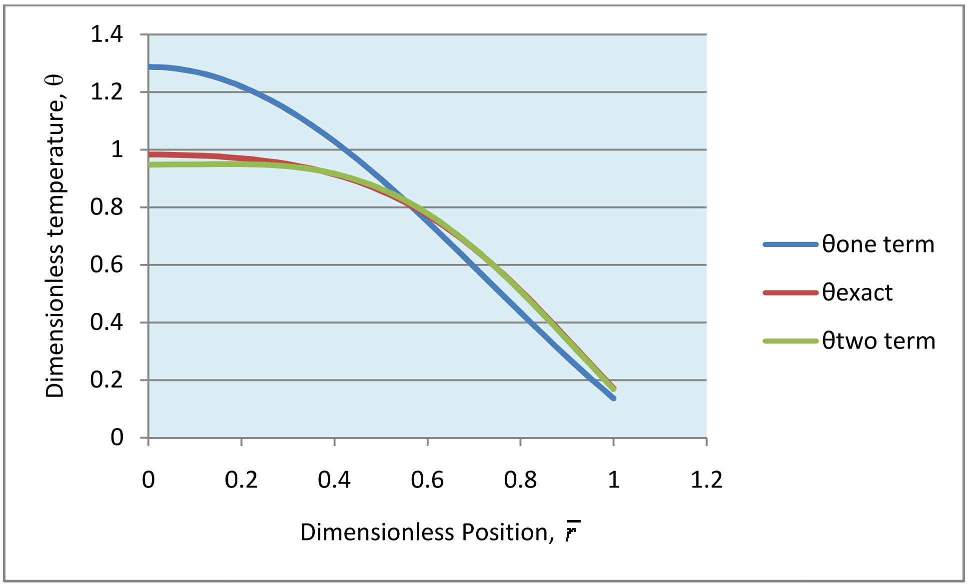

Figure 4.24: Difference between exact solution and approach solution for solid sphere for Bi =10 and τ = 0.05

4.3.1 Analysis of the Error on Two Term Approximation

Using two terms and neglecting all the remaining terms in the series results in small errors relative to one term approximation solution. If the time is more than 0.04, error decreases to the under 1%. It can be seen from Figure 4.25, Figure 4.26, Figure 4.27, Figure 4.28, Figure 4.29 and Figure 4.30. Thus it is very convenient to express the solution using this two term approximation. Two term approximation solution can be used for $\tau > 0.04$ with an average error under 1%. Variation of errors has been shown for three geometries as follows;

Analysis of the error on plane wall

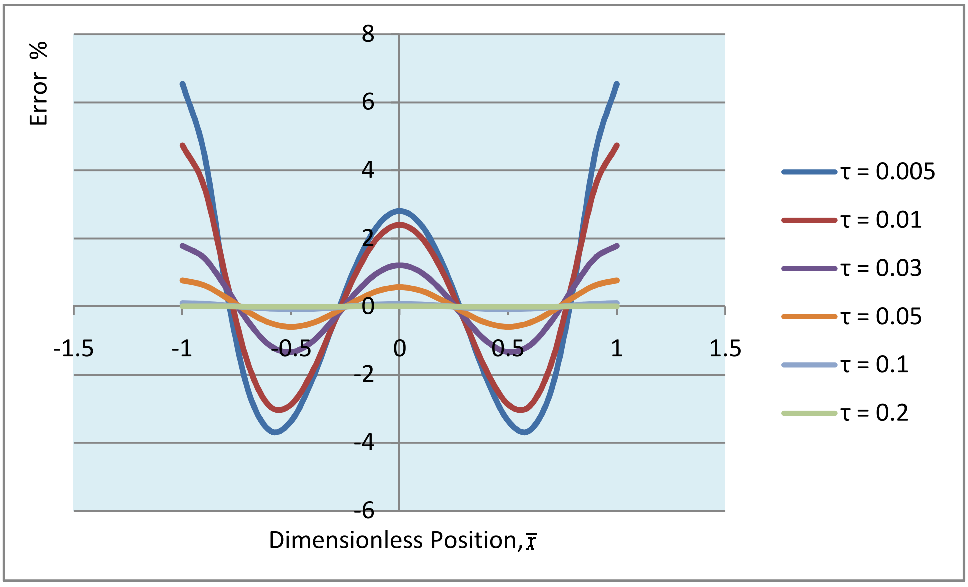

Figure 4.25: Variation of errors for plane wall for Bi = 1

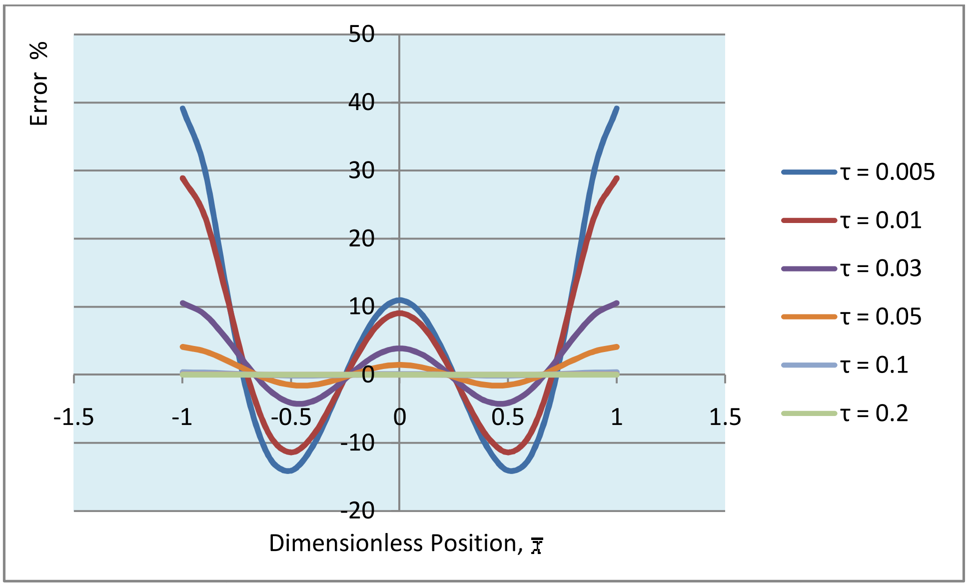

Figure 4.26: Variation of errors for plane wall for Bi = 10

Analysis of the error on long cylinder

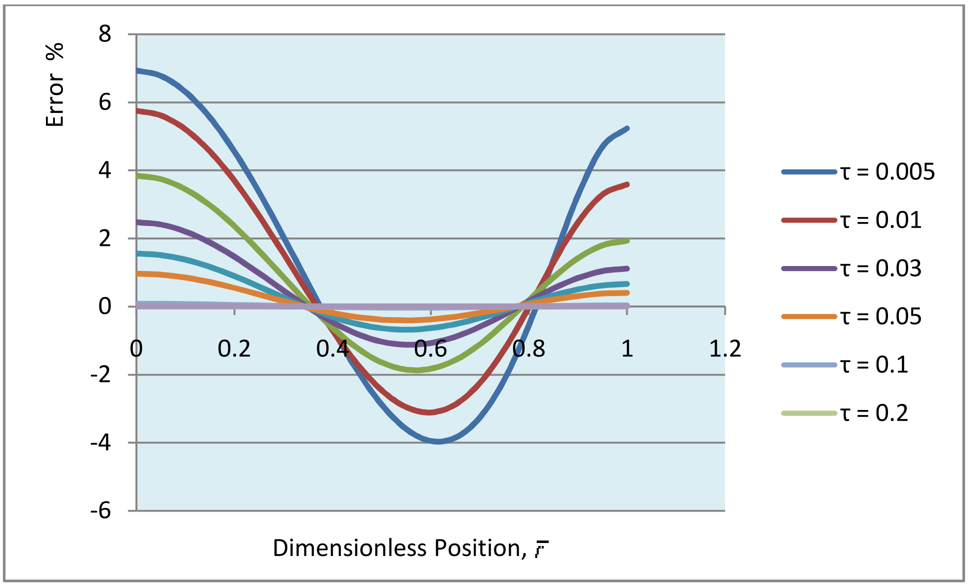

Figure 4.27: Variation of errors for long cylinder for Bi = 1

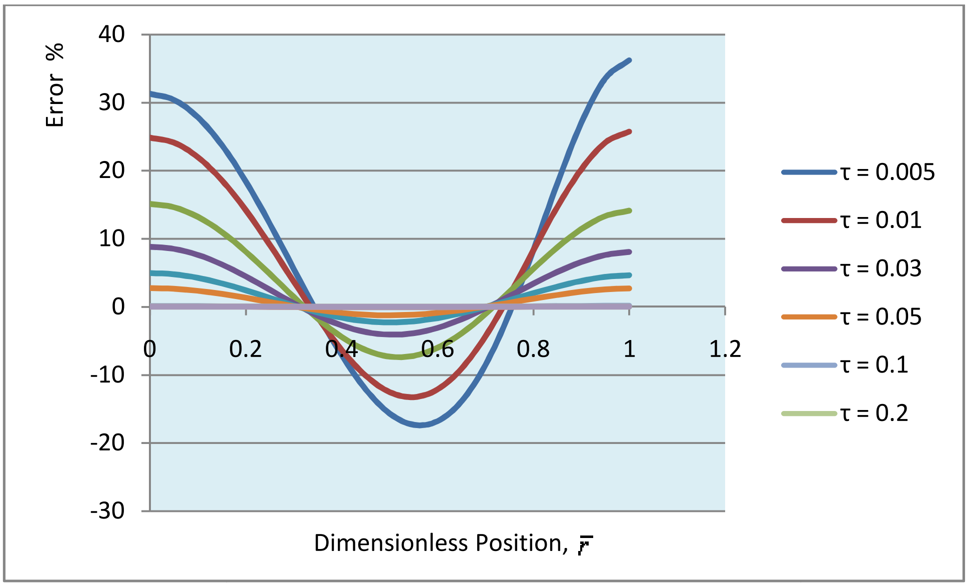

Figure 4.28: Variation of errors for long cylinder for Bi = 10

Analysis of the error on solid sphere

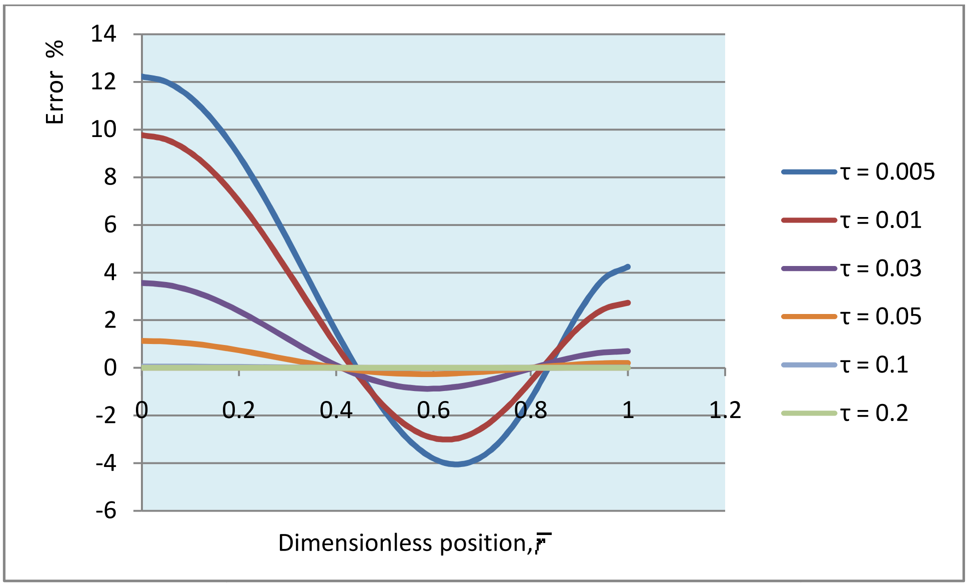

Figure 4.29: Variation of errors for solid sphere for Bi = 1

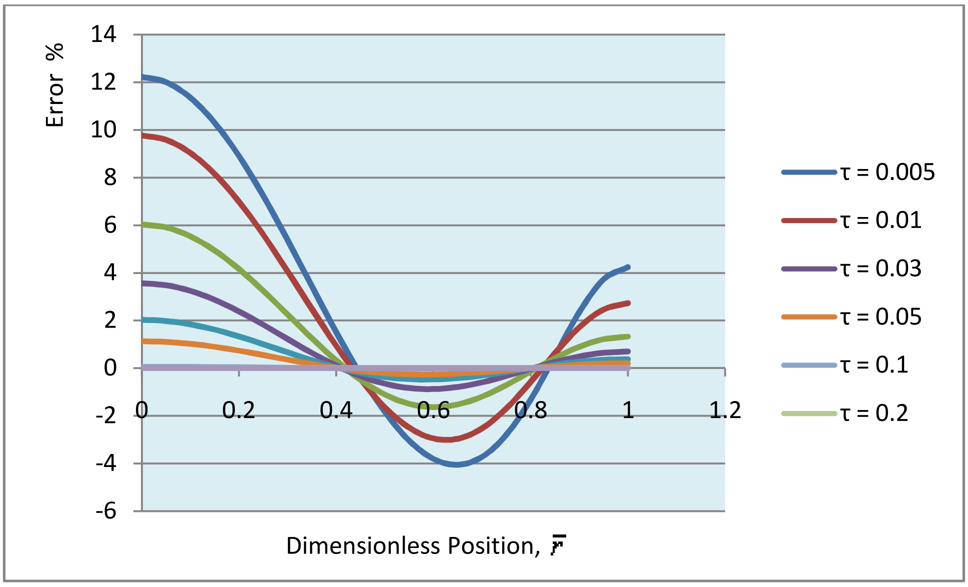

Figure 4.30: Variation of errors for solid sphere for Bi = 10